Abstract

Combinatorics and Complexity in Geometric Visibility Problems

By

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Geometric visibility is fundamental to computational geometry and its applications in areas such as robotics, sensor networks, CAD, and motion planning. We explore combinatorial and computational complexity problems arising in a collection of settings that depend on various notions of visibility.

We first consider a generalized version of the classical art gallery problem in which the input specifies the number of reflex vertices \( r \) and convex vertices \( c \) of the simple polygon \( (n = r + c) \). This additional information better characterizes the ‘shape’ of the polygon. Through a lower bound construction, tight combinatorial bounds for coverage are achieved for all non-negative \( r \) and \( c > 2 \).

The guarding of various polyforms are studied in terms of \( m \), the number of cells, as opposed to the traditional parameter \( n \). Special attention is given to polyominoes, polyforms with unit square cells, where three distinct visibility models are considered. We establish tight combinatorial bounds for covering polyominoes in all models and establish that finding a minimum cardinality guard set is NP-hard. Identical results using point guards are shown to hold for guarding polycubes, connected unions of face-aligned unit cubes.

We consider beacon based point-to-point routing and coverage problems. A beacon \( b \) is a point that can be activated to effect a ‘gravitational pull’ toward itself in a simply connected polygonal domain. We show that finding a minimum cardinality set of beacons to cover a simple polygon or conduct certain types of routing in a simple polygon is NP-hard.

We introduce an algorithm with optimal runtime and space performance for constructing a spiral serpentine polygonization of a set of \( n > 2 \) points in the plane. The algorithm’s behavior can be viewed as incrementally appending a visible triangle to the triangulation constructed so far.

Date: April 27, 2012  
Program: Applied Mathematics and Statistics  
Time: 11:00AM  
Dissertation Advisor: Joseph S. B. Mitchell  
Place: Computer Science, 2311